On the arising of large oscillations with logarithmic wave length in a class of singular perturbation problems with ill-posed limit

New results and examples should be presented in relation with singular perturbation problems of elliptic type involving a small parameter such that, for $\varepsilon > 0$ are well-posed, whereas they are ill-posed at the limit $\varepsilon = 0$ as a consequence of unsuited boundary conditions (specifically, they do not satisfy the Shapiro - Lopatinskii condition). The limit problem has no solution in classical distribution spaces (perhaps it does have in very abstract spaces of analytical functionals not enjoying localization properties). For small values of the solution exhibits large oscillations with moderately small wave length (proportional to $[\log(\varepsilon^{-1})]^{-1}$ along the part of the boundary bearing the "pathological" conditions, exponentially decreasing in the normal direction.

We shall present a new method for handling these problems, using a reduction to a pseudo - differential operator on the boundary. The very structure of the singular behaviour is then apparent, allowing a deeper insight in the structure of the problem and the role of lower order terms. In fact, the presence or not of the above mentioned oscillations is highly dependent of the structure of these terms. More specifically, oscillations are only present when these lower order terms either vanish or have a very precise structure. This question is discussed in relation with physical applications, which are concerned with elliptic shells with a part of the boundary free and with two - dimensional elasticity with very small compression rigidity. In both cases, lower order terms have a special structure, coming from the variational formulation, which allows the mentioned oscillations.