Dimensional reduction for supremal functionals

This talk is devoted to the study of a dimension reduction problem for supremal functionals, which arise naturally as limits of "power law" integral functionals. Specifically, we will be interested in studying the Γ-limit, as $\varepsilon \to 0$, of energy functionals of the form

$$W^{1,\infty}(\Omega_\varepsilon) \ni u \mapsto \operatorname*{ess\,sup}_{x\in\Omega_\varepsilon} W_\varepsilon(x, Du(x)),$$

where $\Omega_\varepsilon := \omega \times (-\varepsilon, \varepsilon)$, $\omega \subset \mathbf{R}^2$ is a bounded open set, and $W_\varepsilon : \Omega_\varepsilon \times \mathbf{R}^3 \to \mathbf{R}^+$ is a Carathéodory supremand. An example of application is the problem of modeling the dielectric breakdown of a thin conductor.
Under mild assumptions on W_ε, we will state an abstract Γ-convergence result and a supremal representation of the Γ-limit. Then we will identify the abstract supremand in some particular cases. When $W_\varepsilon(x, \xi) = W(x, x/\varepsilon, \xi)$, where W is periodic in its second variable, the problem couples dimension reduction and homogenization issues. We will prove in that case that the effective energy density is given by a cell formula which is analogous to that obtained in the integral case.

This is a joint work with F. Prinari and E. Zappale.